Section 10-1, Mathematics 104

Quadratic Equations

Having learned about complex numbers, we are now able to return to solving quadratic equations.

Previously we looked at solving quadratic equations using reverse FOIL factoring:

Example:

$$x^2+5x=24$$
$$x^2+5x-24=0$$
$$(x+8)(x-3)=0$$
$$x=3,-8$$

Completing the square

We talked previously about the completing the square method.

Example:

$$x^2-4x-3=0$$
$$\left(x^2-4x+4\right)-3=4$$
$$(x-2)^2=7$$
$$x-2=\pm\sqrt{7}$$
$$x=2\pm\sqrt{7}$$

The Quadratic Formula

Applying completing the square to the general quadratic equation:

$ax^2 + bx + c = 0$

We get the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example:

$$x^2 + 2x - 1 = 0$$
$$x = \frac{-2 \pm \sqrt{4-(-4)}}{2} = -1 \pm \sqrt{2}$$

The Discriminant

The part of the quadratic formula under the radical sign is called the discriminant.
It determines how many and what kind of solutions there will be.

If $b^2 - 4ac = 0$ then the equation has one real solution.

If $b^2 - 4ac > 0$ then the equation has two real solutions.

If $b^2 - 4ac < 0$ the value under the radical is negative and therefore an imaginary number, so the solutions are both complex. In this case if *b* is zero, then the solutions are pure imaginary.

Examples:

$x^2 + x + 1 = 0$

The discriminant is -3 so there are two complex roots

$x^2 + 6x + 9 = 0$

The discriminant is 36-4x9=0 so there is just one real root

Complex Conjugates

Notice that the two solutions to a quadratic equation are always both complex conjugates.

$$\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)=$$

$$\frac{(-b)^2-\left(\sqrt{b^2-4ac}\right)^2}{4a^2}=\frac{b^2-\left(b^2-4ac\right)}{4a^2}=\frac{4ac}{4a^2}=\frac{c}{a}$$

Examples:

$$x^2-x+2=0$$
$$2x^2-3x-2=0$$
$$x^2-2x+1=0$$
$$x^2-2x-1=0$$

Finding a polynomial equation given its solutions

We know that if $ab = 0$ then either $a = 0$ or $b = 0$.

This gives us a way find a polynomial equations with solutions m and n very easily.

You can see immediately that m and n are solutions to this equation:

$$(x-m)(x-n)=0$$

Example:

Find a quadratic equation for which 3, -4 are solutions?

Using the formula above we have:

$$(x-3)(x-{}^{-}4)=0$$
$$(x-3)(x+4)=0$$
$$x^2+x-12=0$$

Try this one:

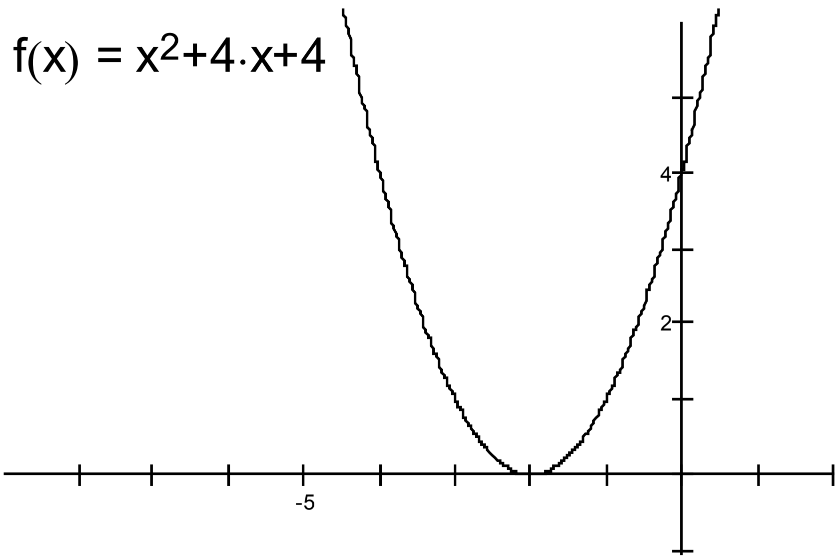
Find a quadratic equation for which 2, 5 are solutions?

How does the discriminant affect the way the graph of a quadratic function?

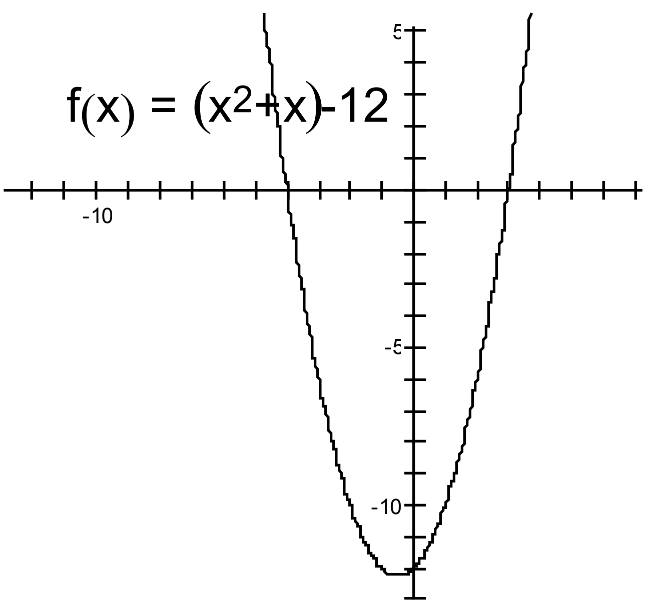
We know that the discriminant tells us how many places the graph of a quadratic function crosses or touches the *x* axis, where *y*=0.

Examples:

$f(x) = x^2 + 4x + 4$ The discriminant is zero so this should touch the *x* axis in just one place:



$f(x) = x^2 + 4x + 4$ The discriminant is > 0 so this should cross the *x* axis in two places:



$f(x) = x^2 + 1$ The discriminant is <0 so this should not cross the x axis: